JII AG

P77

[1] $[1\ 2], [0\ 5], \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}2\\5\end{bmatrix}$

$1, 2, 0, 5$

P78

[2] $\begin{bmatrix}x & 2y\\ z & -6\end{bmatrix} = \begin{bmatrix}-2u & u\\ 4 & 3x\end{bmatrix}$ $\Rightarrow$ $3x = -6$, $\boxed{x=-2}$ $\quad x = -2u$, $-2 = -2u$, $\boxed{u=1}$ $\quad 2y = u$, $2y = 1$, $\boxed{y = \frac{1}{2}}$ $\quad \boxed{z=4}$

P79

[3.1] $\begin{bmatrix}7 & -2 & 4\\ 4 & 2 & -5\end{bmatrix}$

[3.2] $\begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}$

[3.3] $\begin{bmatrix}-2\\ 1\\ 8\end{bmatrix}$

[3.4] $[-5\ \ 2\ \ -6\ \ 8]$

P80

[2.1] $\begin{bmatrix}2 & -9\\ -4 & 5\end{bmatrix}$

[2.2] $\begin{bmatrix}3 & -3 & -5\\ -4 & -4 & 5\end{bmatrix}$

[3] $X = A - B$

$X = \begin{bmatrix}3 & -1\\ -2 & 2\end{bmatrix}$

P81

[4] $3A = \begin{bmatrix}6 & 9 & 12\\ 3 & 0 & -3\end{bmatrix}$

$\frac{1}{2}A = \begin{bmatrix}1 & \frac{3}{2} & 2\\ \frac{1}{2} & 0 & -\frac{1}{2}\end{bmatrix}$

P82

[5]

$$k(A+B) = k\begin{bmatrix} a_{11}+b_{11} & a_{12}+b_{12} \\ a_{21}+b_{21} & a_{22}+b_{22} \end{bmatrix} = \begin{bmatrix} ka_{11}+kb_{11} & ka_{12}+kb_{12} \\ ka_{21}+kb_{21} & ka_{22}+kb_{22} \end{bmatrix}$$

$$kA+kB = \begin{bmatrix} ka_{11} & ka_{12} \\ ka_{21} & ka_{22} \end{bmatrix} + \begin{bmatrix} kb_{11} & kb_{12} \\ kb_{21} & kb_{22} \end{bmatrix} = \begin{bmatrix} ka_{11}+kb_{11} & ka_{12}+kb_{12} \\ ka_{21}+kb_{21} & ka_{22}+kb_{22} \end{bmatrix}$$

[6.1] $3A - B + 2C$

$$= \begin{bmatrix} 6 & 0 \\ -3 & 12 \end{bmatrix} - \begin{bmatrix} 1 & -7 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 10 \\ 16 & -4 \end{bmatrix} = \begin{bmatrix} 5 & 18 \\ 10 & 8 \end{bmatrix}$$

[6.2] $2(A - B + 2C) + 3B - C$

$= 2A - 2B + 4C + 3B - C$

$= 2A + B + 3C$

$$= \begin{bmatrix} 4 & 0 \\ -2 & 8 \end{bmatrix} + \begin{bmatrix} 1 & -7 \\ 3 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 15 \\ 24 & -6 \end{bmatrix} = \begin{bmatrix} 5 & 8 \\ 25 & 2 \end{bmatrix}$$

[7.1] $X + A = 2(2B - X)$

$X + A = 4B - 2X$

$3X = 4B - A$

$X = \frac{4}{3}B - \frac{1}{3}A$

$$= \frac{4}{3}\begin{bmatrix} 5 & -7 \\ 4 & 3 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 2 & -1 \\ 1 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{20}{3} & -\frac{28}{3} \\ \frac{16}{3} & 4 \end{bmatrix} + \begin{bmatrix} -\frac{2}{3} & \frac{1}{3} \\ -\frac{1}{3} & 2 \end{bmatrix}$$

$$X = \begin{bmatrix} 6 & 9 \\ -5 & 6 \end{bmatrix}$$

P 82 CTD

[7,2] $3A + \frac{1}{2}X = X + 4A - \frac{1}{2}B$

$$-\frac{1}{2}X = A - \frac{1}{2}B$$

$$X = -2A + B$$

$$= -2\begin{bmatrix} 2 & -1 \\ 1 & -6 \end{bmatrix} + \begin{bmatrix} 5 & -7 \\ 4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & 2 \\ -2 & 12 \end{bmatrix} + \begin{bmatrix} 5 & -7 \\ 4 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & -5 \\ 2 & 15 \end{bmatrix}$$

P 83

[1,1] $\begin{pmatrix} -1 & 3 \end{pmatrix}\begin{pmatrix} -2 \\ -1 \end{pmatrix} = 2 - 3 = -1$

[1,2] $\begin{pmatrix} \sin\theta & \cos\theta \end{pmatrix}\begin{pmatrix} \sin\theta \\ \cos\theta \end{pmatrix} = \sin^2\theta + \cos^2\theta = 1$

P86

[2.1] $\begin{pmatrix} 1 & -2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 13 & 20 \end{pmatrix}$

$2\times 2 \quad 2\times 2$

[2.2] $\begin{pmatrix} 7 & 5 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} -1 & 2 \\ 0 & 3 \end{pmatrix} = \begin{pmatrix} -7 & 29 \\ -4 & 14 \end{pmatrix}$

$2\times 2 \quad 2\times 2$

2×2

[2.3] $\begin{pmatrix} -2 & 3 \\ 5 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ 4 & 1 \end{pmatrix}$

$2\times 2 \quad 2\times 2$

2×2

[2.4] $\begin{pmatrix} 2 & 3 \\ -3 & 1 \end{pmatrix} \begin{pmatrix} 8 \\ -5 \end{pmatrix} = \begin{pmatrix} 1 \\ -29 \end{pmatrix}$

$2\times 2 \quad 2\times 1$

2×1

[2.5] $\begin{pmatrix} -1 & 1 \end{pmatrix} \begin{pmatrix} 4 & -3 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} -9 & 5 \end{pmatrix}$

$1\times 2 \quad 2\times 2$

1×2

[2.6] $\begin{pmatrix} 2 \\ 3 \end{pmatrix} \begin{pmatrix} -4 & 5 \end{pmatrix} = \begin{pmatrix} -8 & 10 \\ -12 & 15 \end{pmatrix}$

$2\times 1 \quad 1\times 2$

2×2

P88

[1] $A(B+C) = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \left[\begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} \right] = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 4 & 6 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 5 & 13 \\ -5 & 9 \end{pmatrix}$

$AB + AC = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 10 \\ 0 & 10 \end{pmatrix} + \begin{pmatrix} 0 & 3 \\ -5 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 13 \\ -5 & 9 \end{pmatrix}$

[1] $(A+B)C = \left[\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} \right] \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 5 & 5 \\ 0 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} -5 & 5 \\ -10 & -5 \end{pmatrix}$

$AC + BC = \begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} + \begin{pmatrix} 3 & 4 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ -5 & -1 \end{pmatrix} + \begin{pmatrix} -5 & 2 \\ -5 & -4 \end{pmatrix} = \begin{pmatrix} -5 & \\ -10 & \end{pmatrix}$

P 88

[2.1] $AB - AC = A(B-C)$

$$= \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \left[\begin{pmatrix} 4 & 3 \\ 2 & -1 \end{pmatrix} - \begin{pmatrix} -1 & 2 \\ 3 & 4 \end{pmatrix} \right]$$

$$= \underset{2\times 2}{\begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix}} \underset{2\times 2}{\begin{pmatrix} 5 & 1 \\ -1 & -5 \end{pmatrix}} = \begin{pmatrix} -13 & -17 \\ 5 & 1 \end{pmatrix}$$

2×2

[2.2] $AB\,AB = [AB]^2$

$$= \left[\begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ 2 & -1 \end{pmatrix} \right]^2 = \left[\begin{pmatrix} -2 & -9 \\ 4 & 3 \end{pmatrix} \right]^2$$

$$= \begin{pmatrix} -2 & -9 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} -2 & -9 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} -32 & -9 \\ -4 & -27 \end{pmatrix}$$

[2.3] $A^4 = (A^2)^2$

$$A^2 = AA = \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix}$$

$$A^4 = (A^2)^2 = \begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix} \begin{pmatrix} 7 & -6 \\ -2 & 3 \end{pmatrix} = \begin{pmatrix} 61 & -60 \\ -20 & 21 \end{pmatrix}$$

P 89

[3] $\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

[4] $\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

P 90

[5] $\begin{bmatrix} 1 & 1 \\ x & y \end{bmatrix}\begin{bmatrix} 1 & 1 \\ x & y \end{bmatrix} = \begin{bmatrix} 1+x & 1+y \\ x+xy & x+y^2 \end{bmatrix} = 0 \Rightarrow$ $x = -1$, $y = -1$

$\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

[6] NOT Nec True ($AB = AC$ and $A \neq 0$) then $B = C$

{NOTE: In other wards no general cancellation LAW Like for real numbers}

TO show $AB = AC$ AND $A \neq 0$ AND $B \neq C$

Let $A = \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 4 \\ 7 & -1 \end{bmatrix}$, $C = \begin{bmatrix} 5 & -2 \\ 6 & 2 \end{bmatrix}$

Obviously $A \neq 0$ AND $B \neq C$, and

$AB = \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 7 & -1 \end{bmatrix} = \begin{bmatrix} -34 & -4 \\ 17 & 2 \end{bmatrix}$

$AC = \begin{bmatrix} -2 & -4 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 5 & -2 \\ 6 & 2 \end{bmatrix} = \begin{bmatrix} -34 & -4 \\ 17 & 2 \end{bmatrix}$

i.e. $AB = AC$

P 90

[7a] $A^2 = (a+d)A - (ad-bc)E$

$= (1)A - (1)E$

$= \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}$

[7b] $A^3 = A^2 A$

$= \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & -1 \end{bmatrix}$

$A^3 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

P93

$$B = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$BA = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$= \frac{1}{ad-bc}\begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & -bc+ad \end{bmatrix}$$

$$= \frac{1}{ad-bc}\begin{bmatrix} ad-bc & 0 \\ 0 & ad-bc \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

P94

[2,1] $\begin{pmatrix} 0 & 3 \\ 2 & 1 \end{pmatrix} = A$

$$A^{-1} = \frac{1}{0-6}\begin{bmatrix} 1 & -3 \\ -2 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{6} & \frac{1}{2} \\ \frac{1}{3} & 0 \end{bmatrix}$$

[2,2] $\begin{pmatrix} 2 & -1 \\ 3 & 1 \end{pmatrix}$

$$A' = \frac{1}{2+3}\begin{pmatrix} 1 & 1 \\ -3 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} \\ -\frac{3}{5} & \frac{2}{5} \end{bmatrix}$$

p94, ctd

[2.3] $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

$$A^{-1} = \frac{1}{0-1}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$$

[2,4] $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

$A^{-1} = \frac{1}{1-1}$ undefined. No inverse.

p94, ctd

[3]

$$(AB)^{-1} = \left(\begin{bmatrix} 3 & -6 \\ -2 & 5 \end{bmatrix} \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} \right)^{-1} = \begin{bmatrix} 0 & -3 \\ 1 & 5 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{5}{3} & 1 \\ \frac{-1}{3} & 0 \end{bmatrix}$$

$$B^{-1} A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} \frac{5}{3} & 2 \\ \frac{2}{3} & 1 \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & 1 \\ \frac{-1}{3} & 0 \end{bmatrix}$$

[4] $A \neq 0$ and $B \neq 0$ and $AB = 0$ then neither A nor B has an inverse.

Proof (indirect) $\$$ $A \neq 0$, $B \neq 0$, $AB = 0$, and A has inverse A^{-1} and B has inverse B^{-1}.

Since A, B both invertable,

$$AB = 0$$

$$(AB)(AB)^{-1} = 0\,(AB)^{-1}$$

$$I = 0$$

$$\Rightarrow\Leftarrow$$

$\therefore$ claim proved.

P96

[1.1]

$$3x + 2y = 7$$
$$x + 4y = 9$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{12-2} \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 7 \\ 9 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 10 \\ 20 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \end{bmatrix} \qquad \therefore \quad x = 1, \; y = 2$$

[1.2]

$$3x + 2y = 51$$
$$x + 4y = 47$$

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 51 \\ 47 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 51 \\ 47 \end{bmatrix}$$

$$= \frac{1}{12-2} \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} 51 \\ 47 \end{bmatrix}$$

$$= \frac{1}{10} \begin{bmatrix} 110 \\ 90 \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ 9. \end{bmatrix} \qquad \therefore \quad x = 11, \; y = 9$$

P96, ctd

[1.3] $3x + 2y = 89$
$x + 4y = -47$

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 89 \\ -47 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 89 \\ -47 \end{bmatrix}$$

$$= \frac{1}{12-2}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 89 \\ -47 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 450 \\ -230 \end{bmatrix}$$

$$= \begin{bmatrix} 45 \\ -23 \end{bmatrix} \quad \therefore\ x = 45,\ y = -23$$

[1.4] $3x + 2y = 311$
$1x + 4y = 707$

$$\begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 311 \\ 707 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 311 \\ 707 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 311 \\ 707 \end{bmatrix}$$

$$= \frac{1}{10}\begin{bmatrix} -170 \\ 1810 \end{bmatrix}$$

$$= \begin{bmatrix} -17 \\ 181 \end{bmatrix} \quad \therefore\ x = -17,\ y = 181$$

p96 ctd

[2] $$\begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} -1+2t \\ 1+t \end{bmatrix} = \begin{bmatrix} -1+2t-2-2t \\ 3-6t+6+6t \end{bmatrix}$$

$$= \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$\therefore$ Independent of t.

p97

[3] $$\begin{bmatrix} 1 & -2 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

} since $ad-bc = 1\cdot 6 - (-2)(-3) = 0$
Either no solns or inf solns.

$$\begin{bmatrix} x-2y \\ -3x+6y \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \end{bmatrix}$$

$$\begin{aligned} x-2y &= -1 \\ -3x+6y &= 4 \end{aligned} \implies \begin{aligned} x &= -1+2y \\ x &= -\tfrac{4}{3}+2y \end{aligned}$$

$$\implies -1+2y = -\frac{4}{3}+2y$$

$$\implies -1 = -\frac{4}{3}$$

$$\nRightarrow$$

$\therefore$ no soln.

p. 97

[1] $a = 4,\ b = -5,\ c = \frac{5}{6},\ d = 3$

[2.1] $2(A - 3B) - 3(A - C) = -A - 6B + 3C = \begin{pmatrix} -5 & 2 \\ 9 & -4 \end{pmatrix}$

[2.2] $\frac{-1}{3}(2A - 3B + 4C) \Longrightarrow X = \begin{pmatrix} -\frac{5}{3} & -\frac{11}{3} \\ -4 & -1 \end{pmatrix}$

[3] $\text{AB} = (1 \quad -2).\begin{pmatrix} 3 \\ 2 \end{pmatrix} = (\,-1\,)$ and $\text{AC} = (1 \quad -2).\begin{pmatrix} -2 & 1 \\ 4 & 0 \end{pmatrix} = (\,-10 \quad 1\,)$

[4.1] $A^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

[4.2] $A^2 - B^2 = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 3 \\ -4 & 0 \end{pmatrix}$

[4.3]
$$(A + B)(A - B) = \left[\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}\right].\left[\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}\right] = \begin{pmatrix} 2 & 1 \\ -2 & 0 \end{pmatrix}.\begin{pmatrix} 0 & 3 \\ 2 & -2 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 0 & -6 \end{pmatrix}$$

[4.4] $A^2 + 2\,\text{AB} + B^2 = \begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix} + 2\begin{pmatrix} 1 & 2 \\ 0 & -1 \end{pmatrix}.\begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -2 & 1 \end{pmatrix} =$

$$\begin{pmatrix} 1 & 4 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} -6 & 2 \\ 4 & -2 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} -4 & 7 \\ 8 & 0 \end{pmatrix}$$

[5] $\text{LHS} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^2 + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} + \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

$$= \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$\text{RHS} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

Therefore, LHS = RHS.

[6] $(x \quad y).\begin{pmatrix} a & h \\ h & b \end{pmatrix}.\begin{pmatrix} x \\ y \end{pmatrix} = (\,a x + h y \quad h x + b y\,).\begin{pmatrix} x \\ y \end{pmatrix} = (\,a x^2 + 2 h y x + b y^2\,)$

P98

[7.1] Prove $(A+B)' = A' + B'$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$A' = \begin{bmatrix} a & c \\ b & d \end{bmatrix}$, B' $\begin{bmatrix} e & g \\ f & h \end{bmatrix}$

$$LHS = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}'$$

$$= \begin{bmatrix} a+e & c+g \\ b+f & d+h \end{bmatrix}$$

$$= \begin{bmatrix} a & c \\ b & d \end{bmatrix} + \begin{bmatrix} e & g \\ f & h \end{bmatrix}$$

$= R.H.S$

[7.2]

$$LHS = (A^{-1})' = \left(\begin{bmatrix} a & c \\ b & d \end{bmatrix}^{-1}\right)' = \begin{bmatrix} \frac{d}{D} & \frac{-c}{D} \\ \frac{-b}{D} & \frac{a}{D} \end{bmatrix} = \begin{bmatrix} a & c \\ b & d \end{bmatrix}^{-1}$$

$$= (A')^{-1}$$

$$LHS = (A^{-1})' = \begin{bmatrix} \frac{d}{D} & \frac{-b}{D} \\ \frac{-c}{D} & \frac{a}{D} \end{bmatrix}' = \begin{bmatrix} \frac{d}{D} & \frac{-c}{D} \\ \frac{-b}{D} & \frac{a}{D} \end{bmatrix} = \begin{bmatrix} a & +c \\ +b & d \end{bmatrix}^{-1}$$

$$= (A')^{-1}$$

[8] Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, If $a+d=1$ and $ad-bc=0$, then $A^2 = A$.

Proof :

$$A^2 - (a+d)A + (ad-bc)E = 0 \qquad \text{(result p 90)}$$

$$\Rightarrow A^2 - 1A + 0E = 0$$

$$\Rightarrow A^2 - A = 0$$

$$\therefore A^2 = A$$

[9] $A = \begin{bmatrix} 2 & 4 \\ 3 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 10 \\ 5 & 3 \end{bmatrix}$

✓ [9.1] $AX = B$

$$A^{-1}AX = A^{-1}B$$

$$X = A^{-1}B$$

$$= \frac{1}{14-12}\begin{bmatrix} 7 & -4 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} 4 & 10 \\ 5 & 3 \end{bmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 8 & 58 \\ -2 & -24 \end{bmatrix}$$

$$X = \begin{bmatrix} 4 & 29 \\ -1 & -12 \end{bmatrix}$$

[9.2] $X A = B$

$$X A A^{-1} = B A^{-1}$$

$$X = B A^{-1}$$

$$= \begin{bmatrix} 4 & 10 \\ 5 & 3 \end{bmatrix} \frac{1}{2} \begin{bmatrix} 7 & -4 \\ -3 & 2 \end{bmatrix}$$

$$= \frac{1}{2} \begin{bmatrix} -2 & 4 \\ 26 & -14 \end{bmatrix}$$

$$X = \begin{bmatrix} -1 & 2 \\ 13 & -7 \end{bmatrix}$$

[10] $\begin{bmatrix} 2-x & 4 \\ 5 & 3-x \end{bmatrix}$

$$ad - bc = (2-x)(3-x) - (4)(5)$$

$$= 6 - 5x + x^2 - 20$$

$$= x^2 - 5x - 14$$

If $ad - bc = 0$

$$x^2 - 5x - 14 = 0$$

$$(x-7)(x+2) = 0$$

$$x = 7 \text{ or } x = -2$$

$\therefore$ If $x = 7$ or $x = -2$, matrix has no inverse.